# How important is to include self-consistent magnetic field in inner magnetosphere modeling

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# Inner Magnetosphere Particle Transport and Acceleration Model

**(***Ganushkina et al., AnnGeo, 2005; JGR, 2006; AnnGeo, 2012***)**

- Changes in distribution function f and flux calculations for ions and electrons **with arbitrary pitch angles** using ***Liouville*'s theorem** taking into account **loss processes**.

$$\frac{df}{dt} = \frac{\partial f}{\partial \phi} \cdot V_\phi + \frac{\partial f}{\partial r} \cdot V_r + sources - losses$$

- **Boundary distribution**: at any location from 6.6 to 10 Re
- **Transport of particles**:
  - -Drifts with velocities, radial and longitudinal, as sum of **ExB and magnetic drifts, 1**st and 2nd inv = const in **time-dependent magnetic and electric fields** with self-consistent magnetic field

$$\mathrm{Vdrift} = \frac{\vec{\mathrm{E}} \times \vec{\mathrm{B}}}{\mathrm{B}^2} + \frac{\mathrm{mv}_\perp^2}{2\mathrm{qB}^3}\left(\vec{\mathrm{B}} \times \nabla \mathrm{B}\right) + \frac{\mathrm{mv}_{\mathrm{II}}^2}{\mathrm{q}} \frac{\vec{\mathrm{R}}_\mathrm{c} \times \vec{\mathrm{B}}}{\mathrm{R}_\mathrm{c}^2 \mathrm{B}^2}$$

$$\langle v_0 \rangle = \frac{\mathbf{E}_0 \times \mathbf{B}_0}{B_0^2} + \frac{2p}{q\tau_b B_0} \nabla I \times e_0,$$

$$I = \int_{S_m}^{S_m'} \left[1 - B(s)/B_m\right]^{1/2} ds,$$

# Inner Magnetosphere Particle Transport and Acceleration Model: Diffusion

Next **Radial diffusion** is applied (*Schulz and Lanzerotti*, 1974)

$$\frac{df}{dt} = L^2 \frac{\partial}{\partial L}\left(\frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau}$$

with diffusion coefficients $D_{LL}$ (*Brautigam and Albert*, 2000)

$$D_{LL} = 10^{0.056Kp - 9.325} L^{10}$$

And **Pitch- angle diffusion** by introducing electron lifetimes

- by *Chen et al*. (2005) for strong diffusion
- and *Shprits et al*. (2007) for weak diffusion

# Inner Magnetosphere Particle Transport and Acceleration Model: Electrons' Lifetimes

**Strong diffusion**: $$\tau_{sd} = \left(\frac{\gamma m_0}{p}\right)\left[\frac{2\Psi B_h}{1-\eta}\right]$$

p is the particle momentum, γ is the ratio of relativistic mass to rest mass, Bh is the magnetic field at either foot point of field line, Ψ is the magnetic flux tube volume, η =0.25 backscatter coefficient (25% of electrons that will mirror at or below 0.02 Re are scattered back to flux tube instead of precipitating into atmosphere)

**Weak diffusion**: $$\tau_{wd} = 4.8 \cdot 10^4 B_w^{-2} L^{-1} E^2, \quad B_w^2 = 2 \cdot 10^{2.5+0.18Kp}$$

Bw is the local wave amplitude, E is kinetic energy in MeV

# Inner Magnetosphere Particle Transport and Acceleration Model: Losses

**Losses for ions:**

- **charge exchange** with Hydrogen from geocorona;
- **Coulomb interaction** in dense thermal plasmas (plasmasphere);
- **convection outflow**, particle intersects the magnetopause and flows away along magnetosheath magnetic field lines.

**Losses for electrons:**

- **Coulomb collisions** and loss to the atmosphere;
- **convection outflow**, particle intersects the magnetopause and flows away along magnetosheath magnetic field lines;
- scattering into the loss cone due to **pitch angle diffusion**.

# Inner Magnetosphere Particle Transport and Acceleration Model: Boundary Conditions

Boundary distribution at 10 $R_E$ is kappa function:

$$f(E) = n\left(\frac{m}{2\pi k E_0}\right)^{3/2} \frac{\Gamma(k+1)}{\Gamma(k-1/2)}\left(1+\frac{E}{kE_0}\right)^{-(k+1)},$$

where n is the particle number density, m is the particle mass, $E_0 = k_B T(1-3/2k)$ is the particle energy at the peak of the distribution, $k_B$ is the Boltzmann constant, and $T = 1/3(T_\parallel + 2T_\perp)$ The gamma functions were computed for k=5.

Ion temperature and number density is given *by Tsyganenko and Mukai* (2003) model. The electron number density the same as for ions, for electron temperature the correction factor Te/Ti = 0.2 taken into account (*Kaufmann et al*. [2005]; *Chih-Ping Wang*, based on Geotail data, private communication, 2010).

# Inner Magnetosphere Particle Transport and Acceleration Model: Ion species

1. **At 6.6 Re, LANL observations**

Scaling of the observed number density following *Young et al.*, 1982

n(H+) = 0.34 * exp(0.054 * Kp)

n(He+) = 0.0051 * exp(0.0066 * F10.7)

n(O+) = 0.011*exp(0.24 * Kp + 0.011 * F10.7)

2. **At 10 Re, *Tsyganenko and Mukai* [ 2003] model**

Scaling of the observed number density following *Mouikis et al.*, 2010

n(H+) = 0.19 * exp(0.002*F10.7 + 0.078 *(Kp - 1))

n(O+) = 0.002 * exp(0.008 * F10.7 + 0.43 *(Kp -1))

# Including self-consistent magnetic field

- Obtain parallel pressure and perpendicular pressure from IMPTAM

$$P_{\parallel} = \int mv^2 f \cos^2 \alpha dp, \quad P_{\perp} = \int \frac{1}{2} mv^2 f \sin^2 \alpha dp, \quad dp = m^3 v^2 d\Omega dv$$

- Calculate the current perpendicular to magnetic field

$$\vec{j}_{\perp} = \frac{\vec{B}}{B^2} \times \left( \nabla P_{\perp} + \frac{P_{II} - P_{\perp}}{B^2} \left( \vec{B} \cdot \nabla \right) \vec{B} \right)$$

- Calculate the magnetic field induced by the ring and near-Earth tail currents using the Biot-Savart law

$$B(\vec{r}) = \frac{\mu_0}{4\pi} \int \int \int \frac{\vec{J}_{\perp}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{\left| \vec{r} - \vec{r}\,' \right|^3} d^3 r'$$

- Calculated magnetic field is then used in IMPTAM to update the particle trajectories

- The procedure repeated 2 or 3 times, dependent on when the following calculations do not differ from the previous ones

# Model-dependent Dst calculations during storms

1. Using **Dessler-Parker-Sckopke relationship**:

The energy in the ring current can be expressed by $\frac{\Delta\vec{B}}{B_E} = -\frac{2}{3}\frac{W_{RC}}{W_{mag}}\hat{k}$, where

$W_{mag} = \frac{4\pi}{3\mu_0}B_E^2 R_E^3$ is the total energy in the Earth's dipole magnetic field above the surface, $B_E$ is the magnetic field at the Earth's surface, $R_E$ is one Earth radii (6371 km).

$\Delta\vec{B}$ is the change in B measured at the surface of the Earth (Dst).

**2. Calculating from the model ring current by Biot-Savart law:**

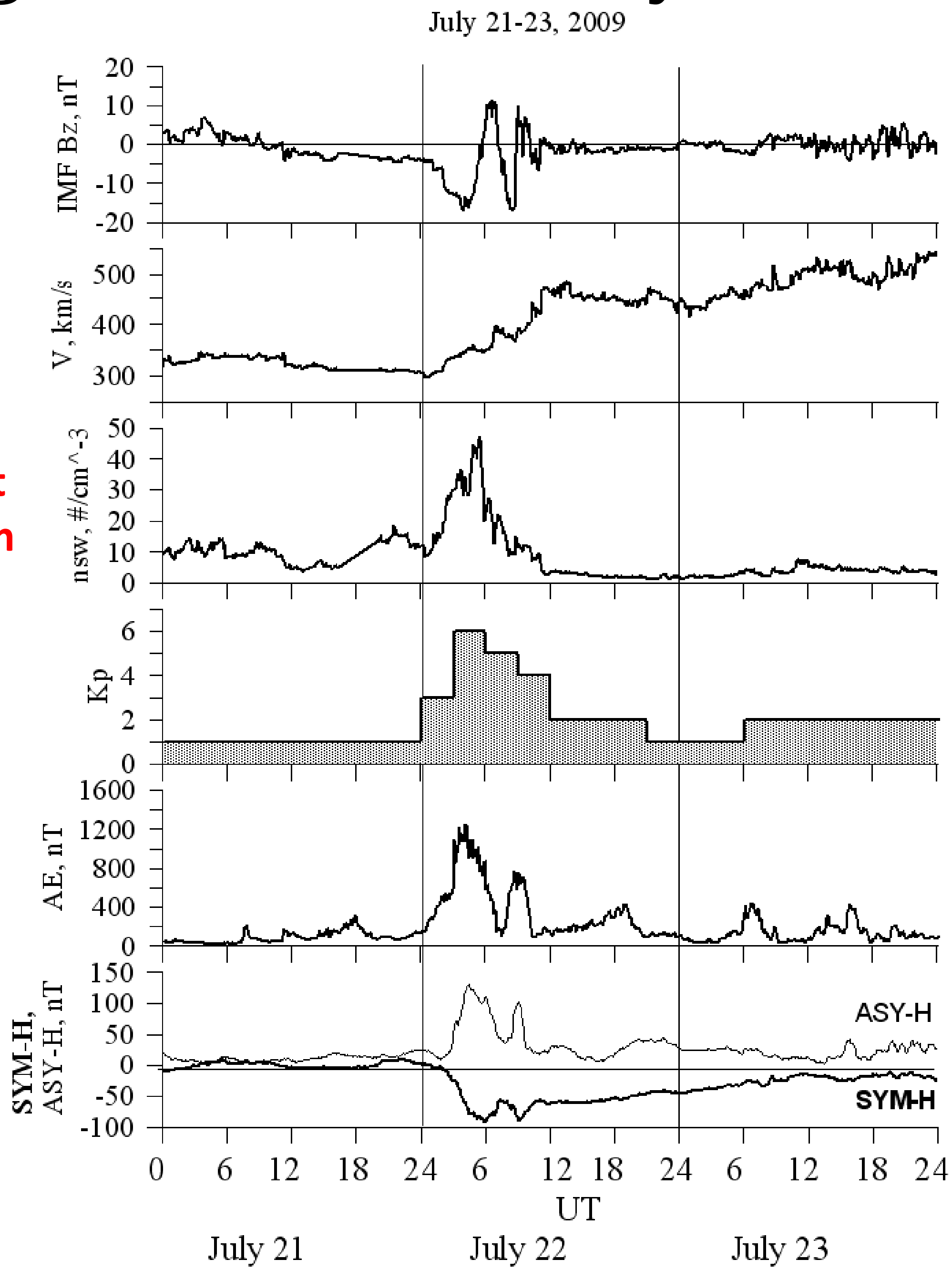
The magnetic disturbance parallel to the earth's dipole at the center of the earth $\Delta B$ induced by the azimuthal component of $J_\perp$, is given by

$$\Delta B = \frac{\mu_0}{4\pi}\int_r \int_\lambda \int_\phi \cos^2\lambda J_\phi(r,\lambda,\phi)\,dr\,d\lambda\,d\phi$$

$$\vec{j}_\perp = \frac{\vec{B}}{B^2}\times\left(\nabla P_\perp + \frac{P_{II} - P_\perp}{B^2}(\vec{B}\cdot\nabla)\vec{B}\right)$$

# Magnetic storm on July 21-23, 2009

Smooth southward turning of IMF Bz




High speed stream

Density peak in front of High Speed stream

Substorm activity

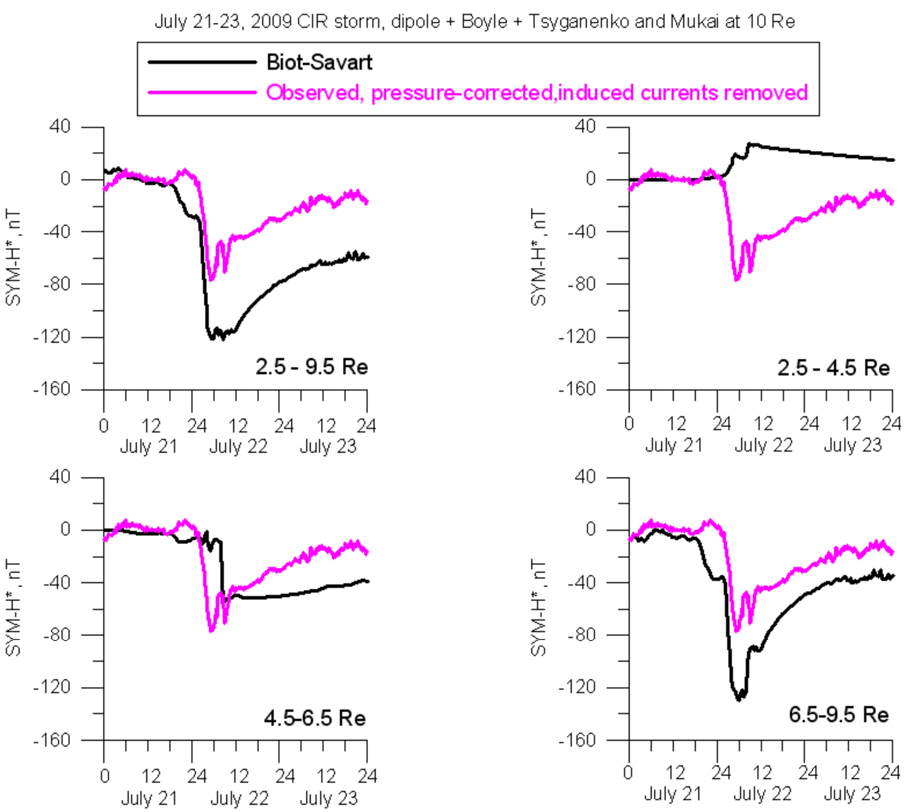
Small storm

Long recovery

# Modeled Dst for  July 21-23, 2009 storm

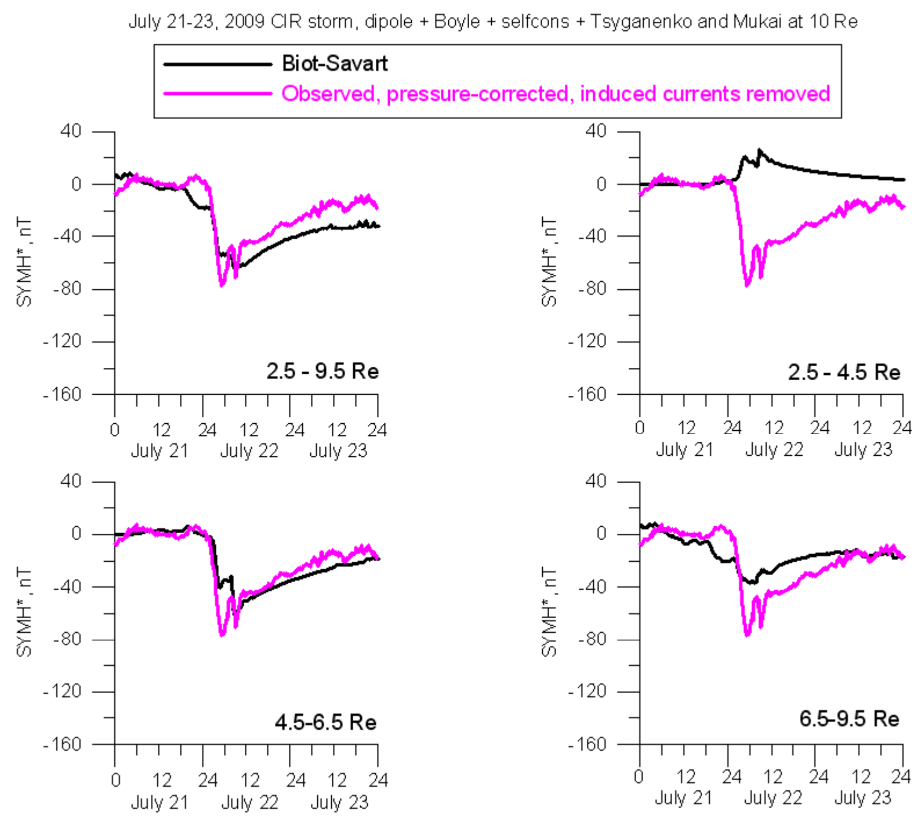
## Dip + Boyle + Tsyganenko and Mukai, 2003 at 10 Re

### without self-consistent mag. field




Storm maximum:
Underestimate of total model SYM-H by 40 nT
Main contribution from 6.5-9.5 Re (tail)

### with self-consistent mag. field

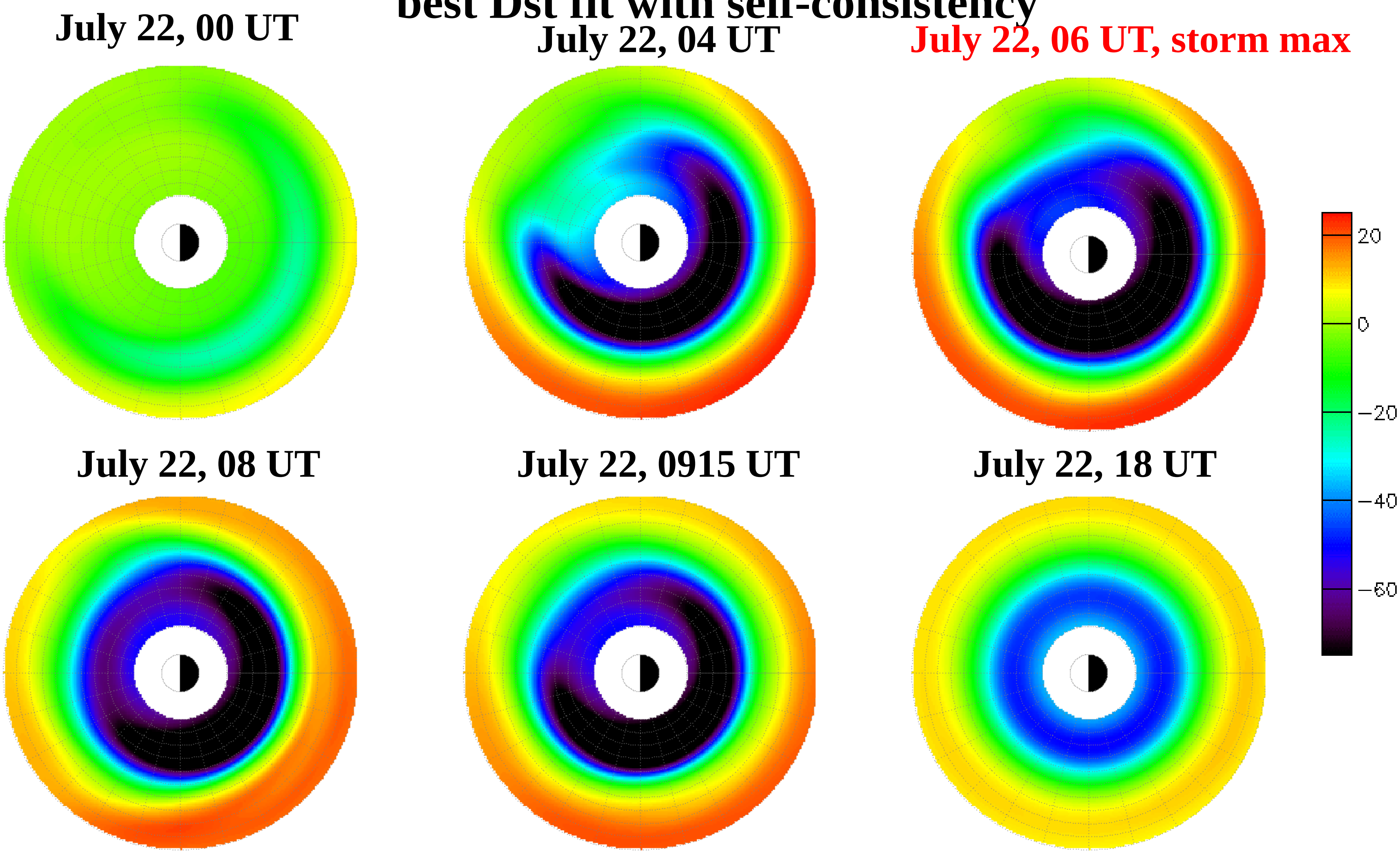



Total model SYM-H comparable with obs
4.5-6.5 Re (ring) no change,
6.5-9.5 Re (tail) overestimate by 40 nT

# Induced magnetic field for July 21-23, 2009 storm,

## Dip + Boyle + Tsyganenko and Mukai, 2003 at 10 Re,

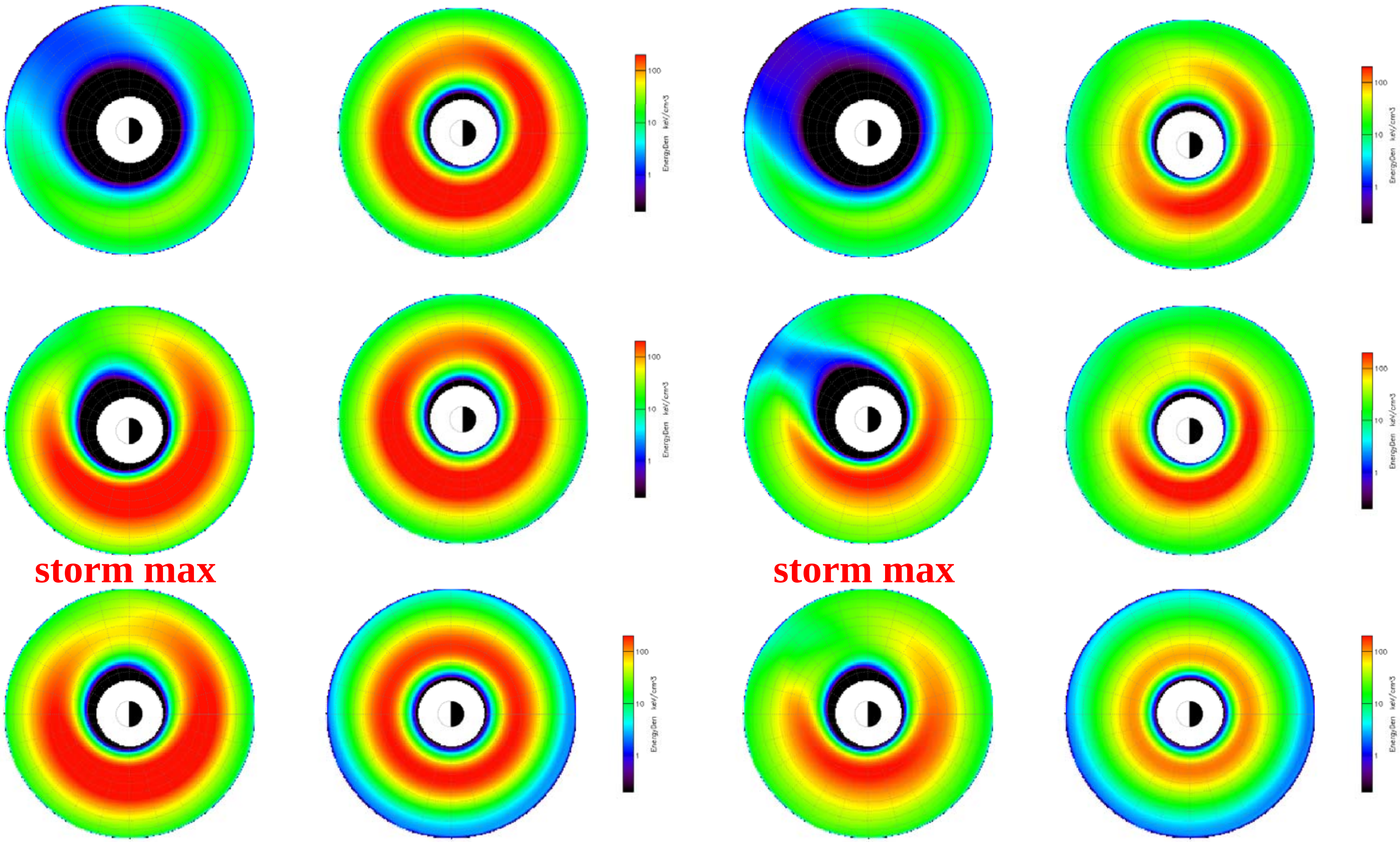
## best Dst fit with self-consistency



# Equatorial energy density maps for July 21-23, 2009 storm

## Dip + Boyle + Tsyganenko and Mukai, 2003 at 10 Re

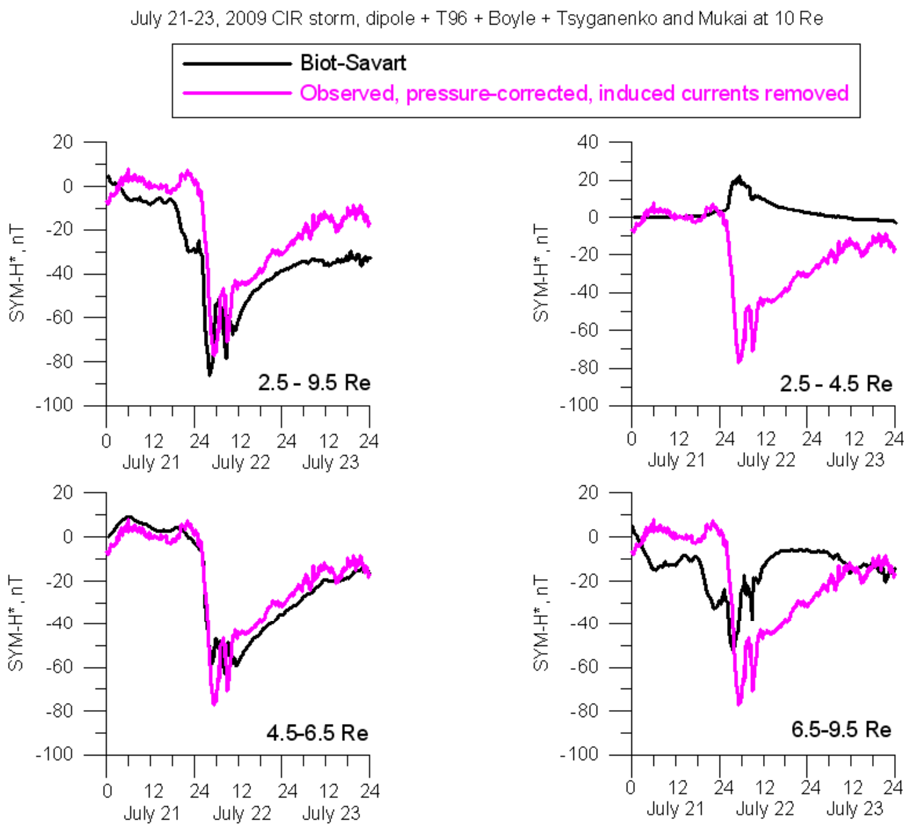
**without self-consistent mag. field** **with self-consistent mag. field**



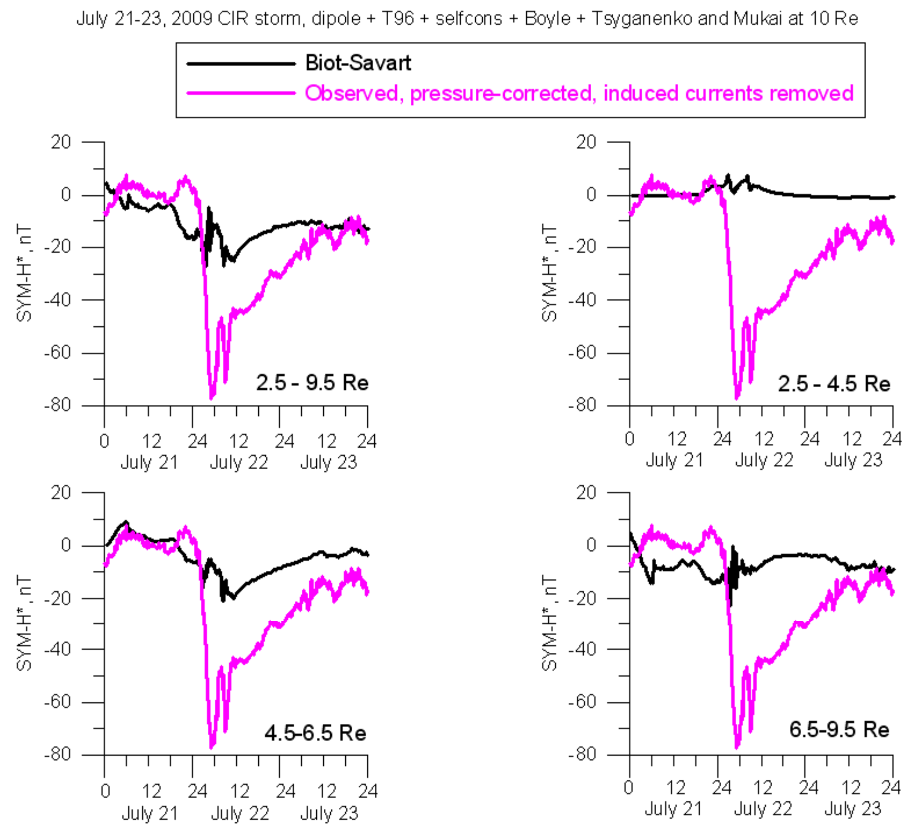
# Modeled Dst for July 21-23, 2009 storm

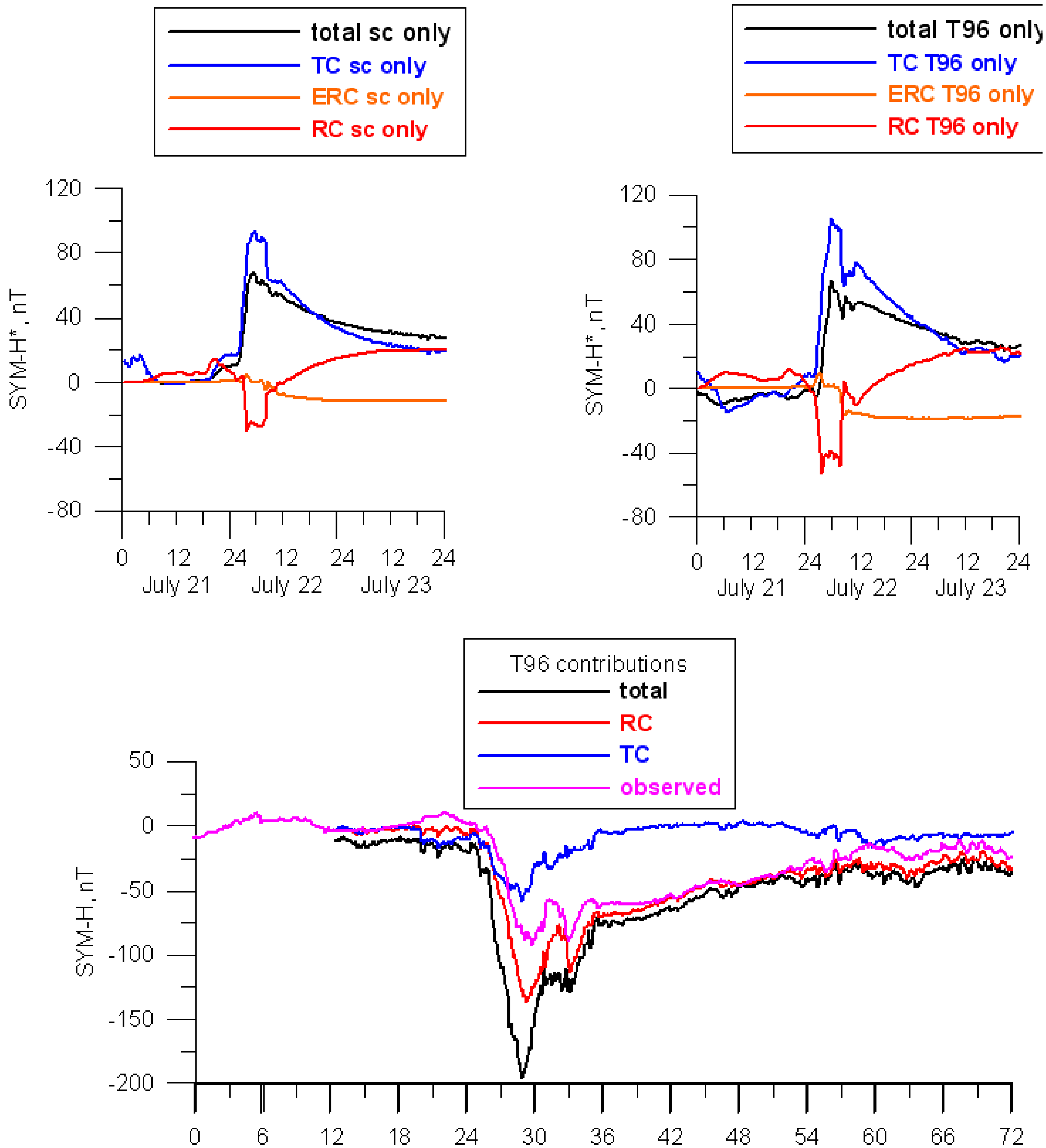
## Dip + T96 + Boyle + Tsyganenko and Mukai, 2003 at 10 Re

### without self-consistent mag. field




### with self-consistent mag. field



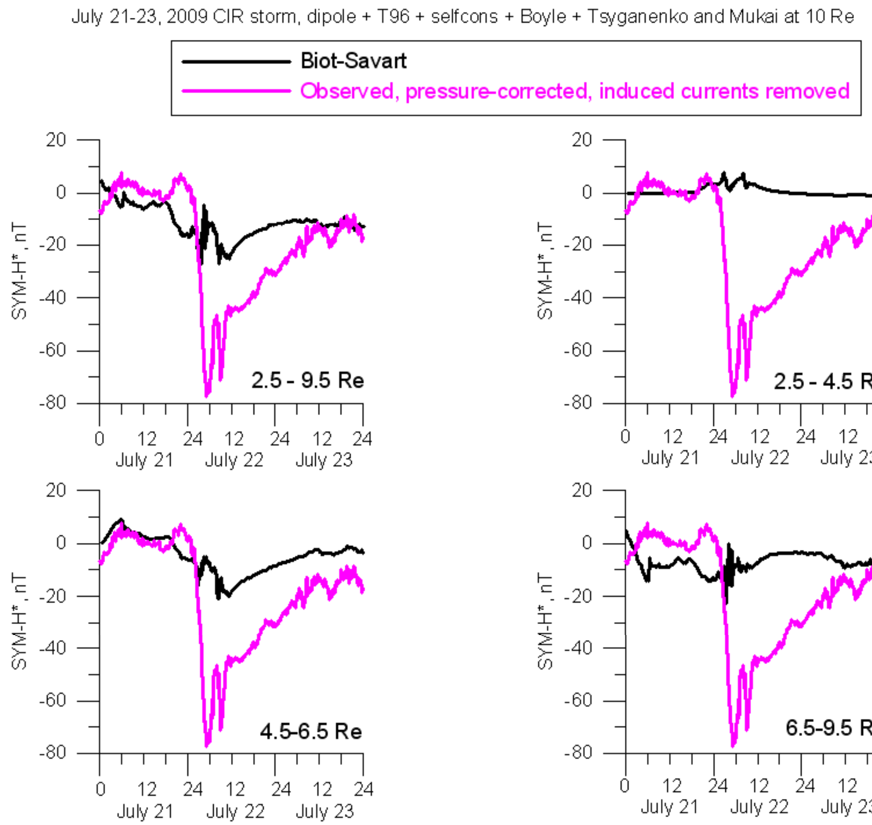


total sc only
TC sc only
ERC sc only
RC sc only
SYM-H*, nT
120
80
40
0
-40
-80
0
12
24
12
24
12
24
July 21
July 22
July 23
total T96 only
TC T96 only
ERC T96 only
RC T96 only
SYM-H*, nT
120
80
40
0
-40
-80
0
12
24
12
24
12
24
July 21
July 22
July 23
T96 contributions
total
RC
TC
observed
SYM-H, nT
50
0
-50
-100
-150
-200
0
6
12
18
24
30
36
42
48
54
60
66
72

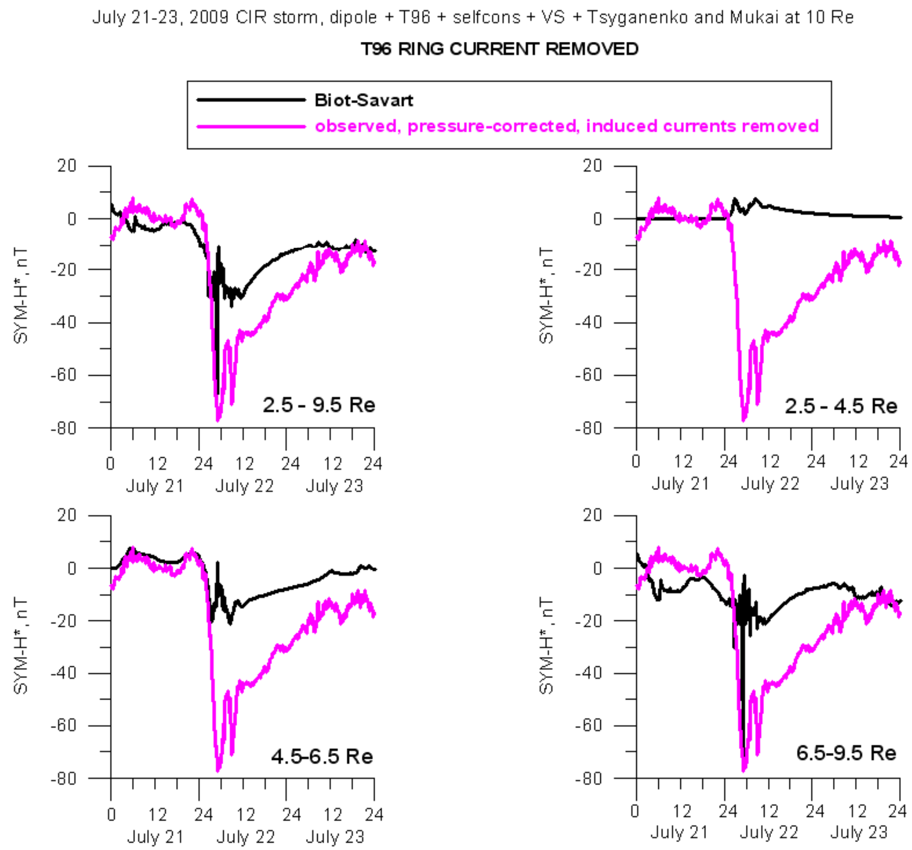
# Modeled Dst for July 21-23, 2009 storm

## Dip + T96 + Boyle + Tsyganenko and Mukai, 2003 at 10 Re

### with self-consistent mag. field



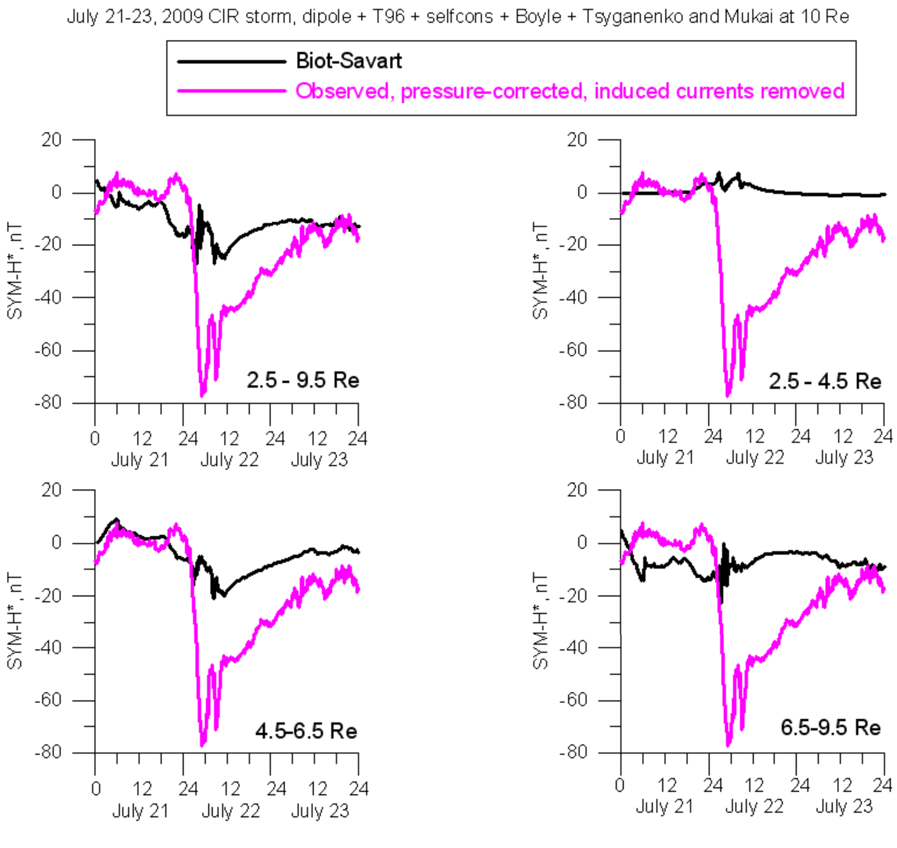

### T96 RC removed



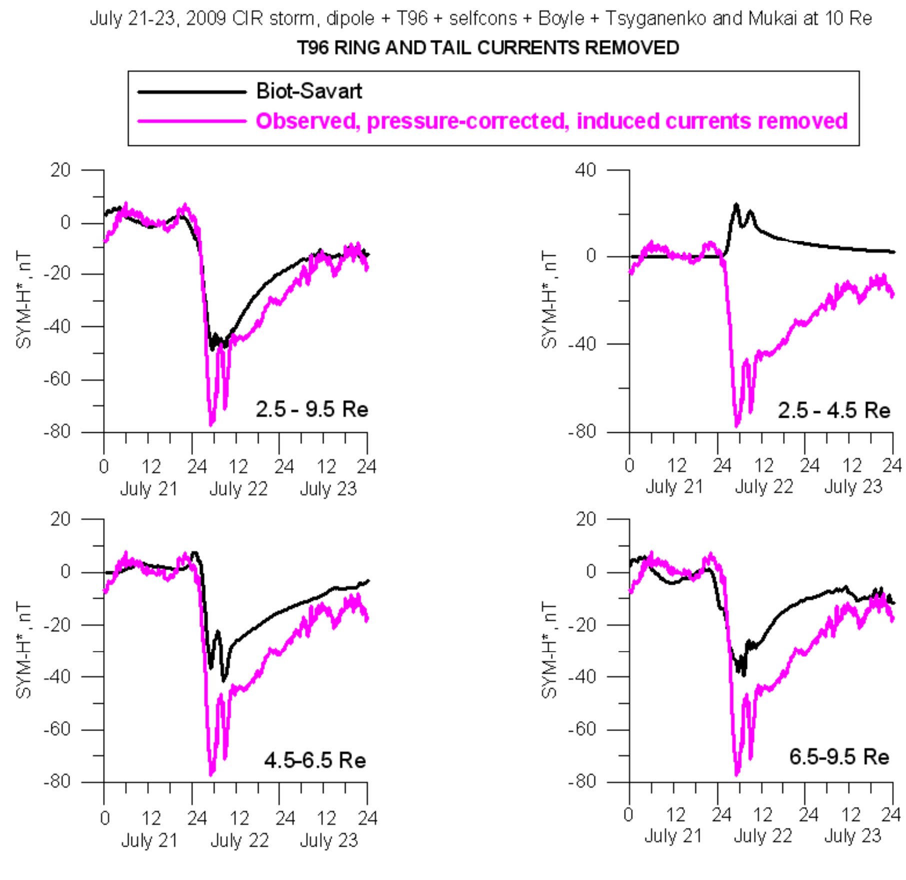
# Modeled Dst for July 21-23, 2009 storm

## Dip + T96 + Boyle + Tsyganenko and Mukai, 2003 at 10 Re

### with self-consistent mag. field




### T96 RC and TC removed



# Summary

1. Including **self-consistent magnetic field** when modeling inner magnetosphere **in dipole background** magnetic field and
modeling **in T96 magnetic field** model give comparable results

Can we just use realistic magnetic field models and not include self-consistency?

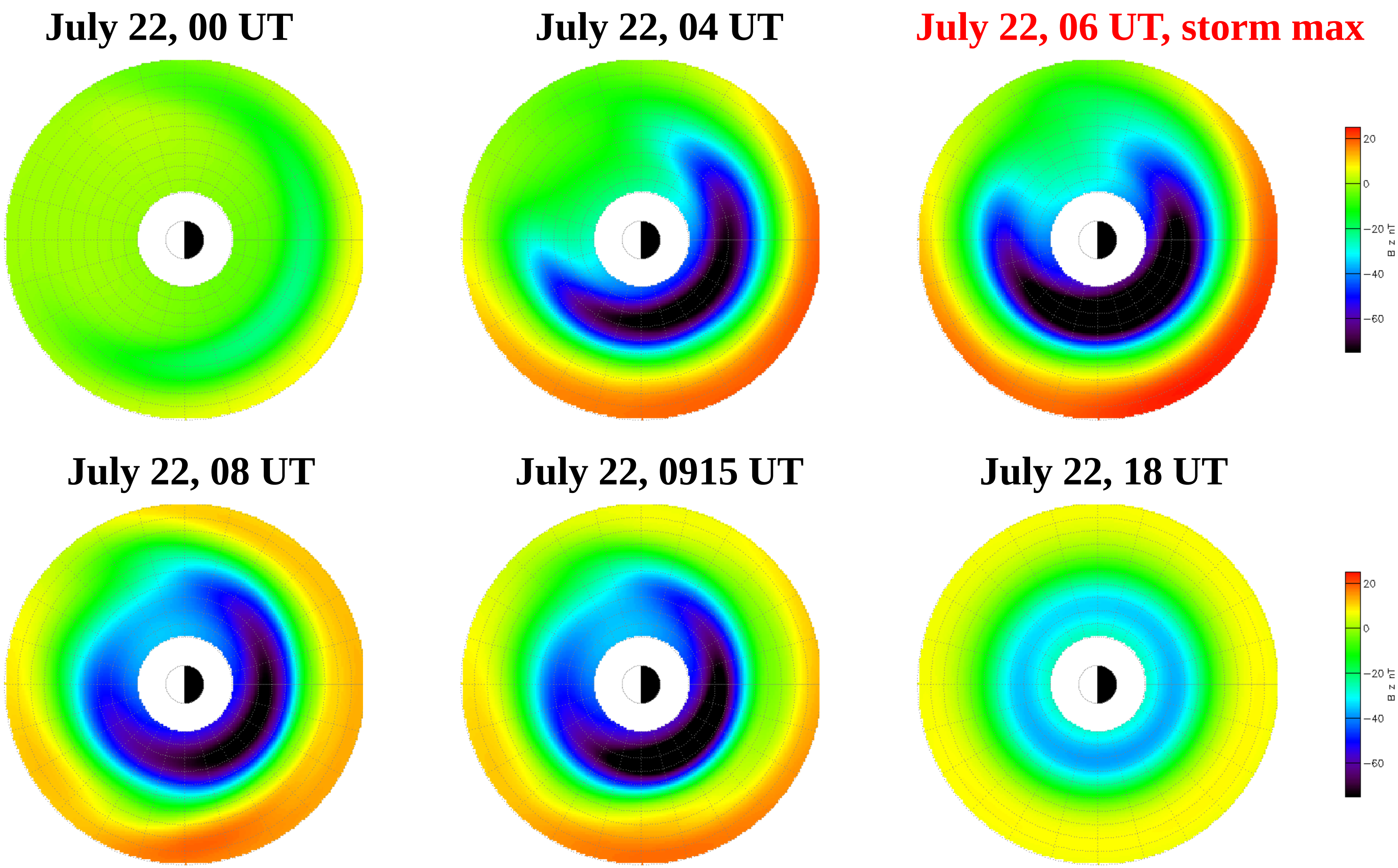
2. Including self-consistent magnetic field when modeling with T96 background magnetic field model but with T96 ring and tail currents removed

Possible way to include self-consistency to realistic magnetic field models?

# Induced magnetic field for July 21-23, 2009 storm

## Dip + T96 – RC, TC removed + VS + Tsyganenko and Mukai, 2003 at 10 Re

## best Dst fit without self-consistency (with T96 RC and TC)



**Equatorial energy density maps for July 21-23, 2009 storm**

**Dip + T96 + VS + Tsyganenko and Mukai, 2003 at 10 Re**

**without self-consistent mag. field with self-cons. mag. f.**

**T96 RC and TC removed**